Zombies and Differential Equations

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Topics Covered

- Basic Predator-Prey Equation
- Basic Equation Adjusted for Zombies
- Basic Equation with Latency
- The Quarantine Model
- The Treatment Equation
- Intelligent Human Resistance

Introduction

The fear of the living dead, or zombies, has probably been around since the first group of humans witnessed someone dying. The thought of someone dying, then coming back to life as "soul-less" object is terrifying. There has never been any evidence of a zombie or zombie outbreak, but just to be safe, we will be detailing how the spread of zombies will affect your ability to survive or be killed through a series of differential equations.

Basic Equation

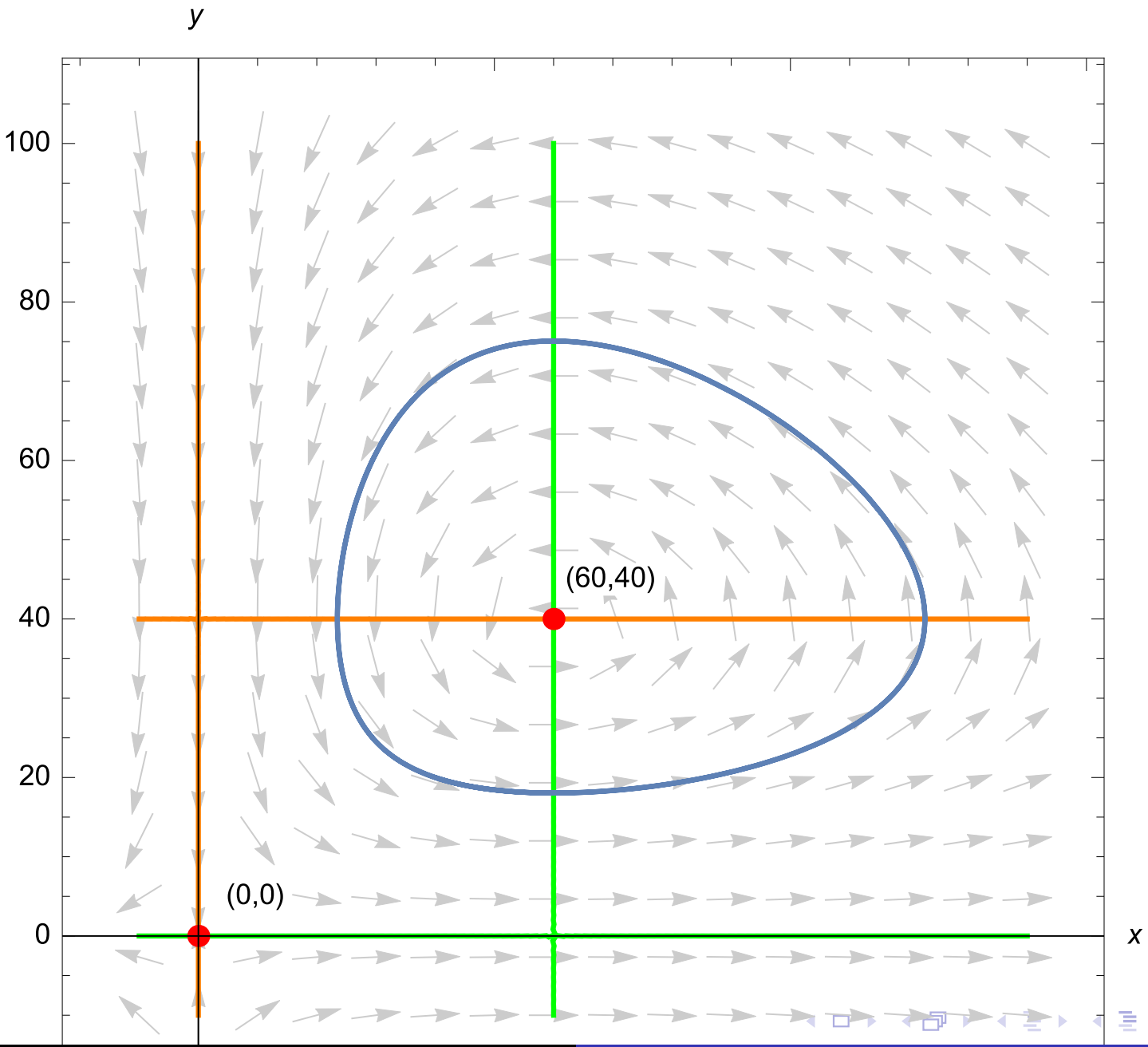
The following is the Lotka-Volterra predator-prey equation. This equation shows the relationship between humans and zombies with a basic predator prey situation.

$$X' = \alpha X - \beta XY$$

$$Y' = \delta XY - \gamma Y$$

X= Prey
Y= Predators
α= The birthrate of the prey.
β = The death rate of the prey from the predator.
γ = The death rate of the predators.
δ = The growth rate of the predators based on their food supply (prey).
X' and Y'= The change in population with respect to time, respectively.

Here, we see a basic prey relationship plotted.



Basic Equation Adjusted for Zombies

The predator-prey equation is great for predator prey relationship, but In our situation, once a human (the prey) dies, they will turn into a zombie (predator). Our new "Basic Equation" will become:

$$H' = \chi H - \gamma HZ - \zeta H$$
$$Z' = \gamma HZ + \delta D - \kappa HZ$$
$$D' = \zeta H + \kappa HZ - \delta D$$

H = Humans, Z = Zombies, D =Dead
δ = The rate at which humans can re-animate as zombies (.02).
ζ = Human deaths through natural causes (.00596).
γ = Human conversion rate to zombies through encounters (varies).
χ = Birth rate of humans (1%) .
κ = Rate at which humans kill zombies (varies).

Here, we see the result of a zombie takeover.

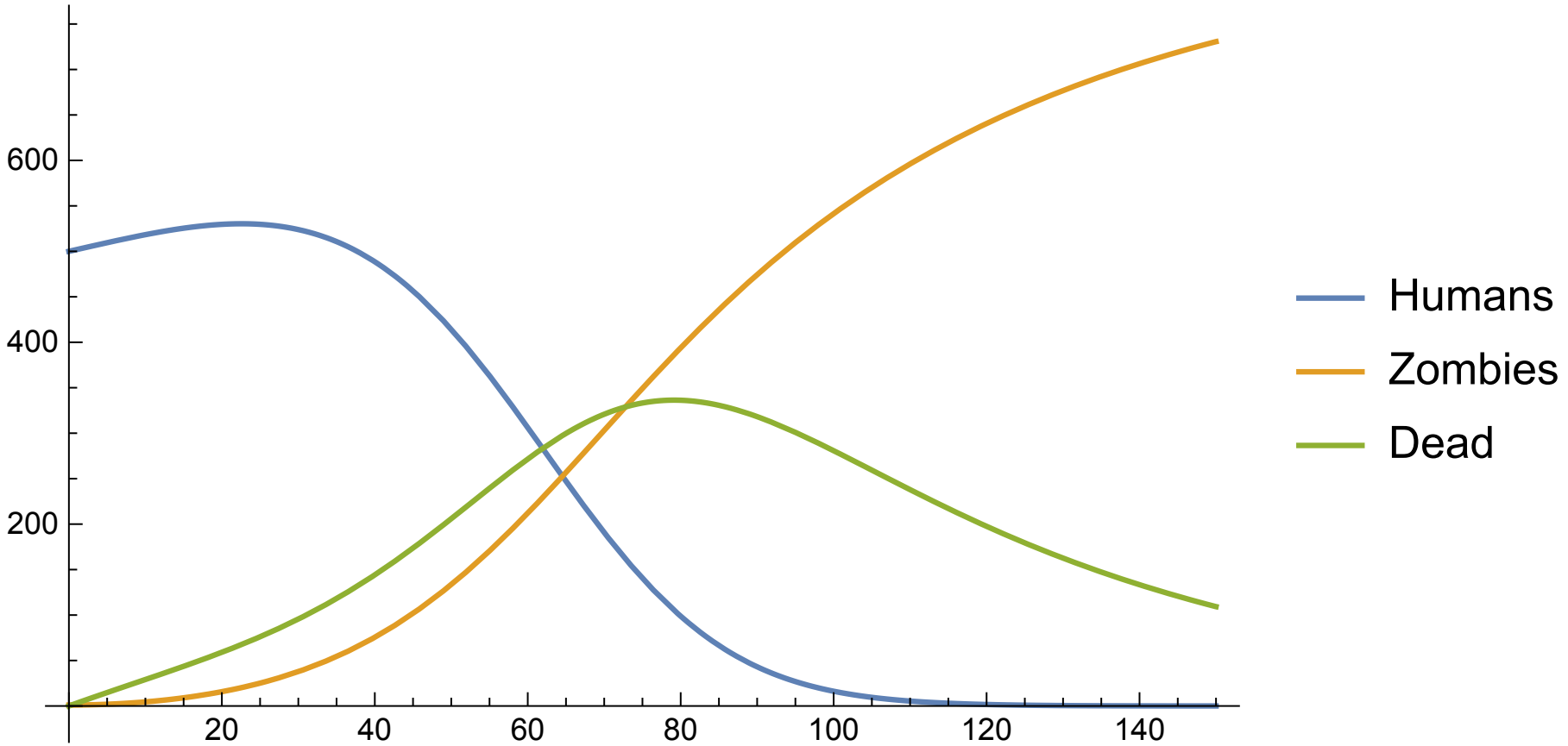


Figure: Zombies More Effective

$\kappa = .00015$, while $\gamma = .0002$.

Zombies still take over, despite the humans being better at killing zombies.

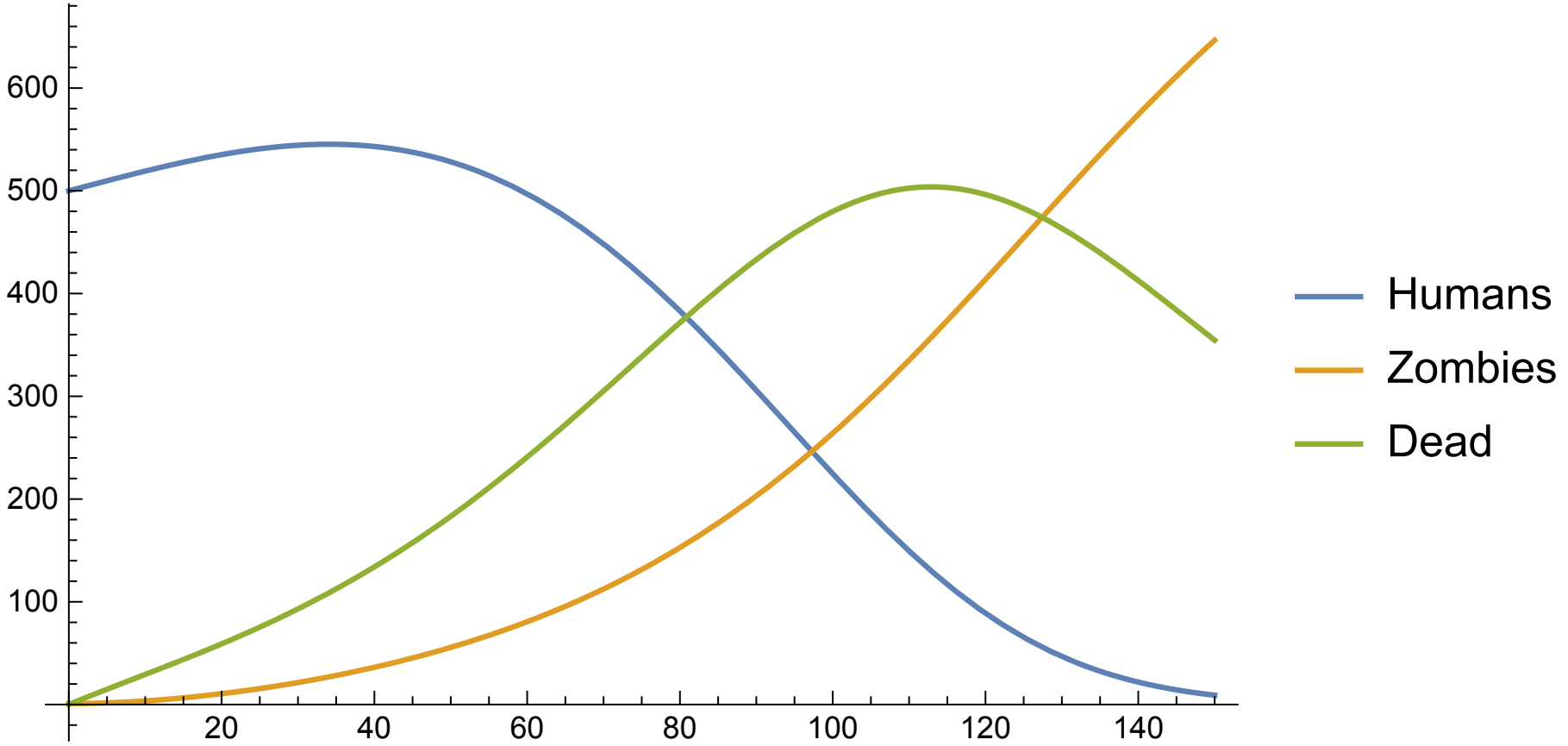


Figure: Humans More Effective

$\kappa = .0002$, and $\gamma = .00015$.

Basic Model with Latency

- Latency: ability of a virus or infection to stay inside an individual without showing any symptoms until a duration of time has passed.

$$H' = \chi H - \gamma HZ - \zeta H$$
$$I' = \gamma HZ - \phi I - \zeta I$$
$$Z' = \phi I + \delta D - \kappa HZ$$
$$D' = \zeta H + \zeta I + \kappa HZ - \delta D$$

I= Infected individuals
ϕ= Rate at which the I class dies (goes into D class). (.006).

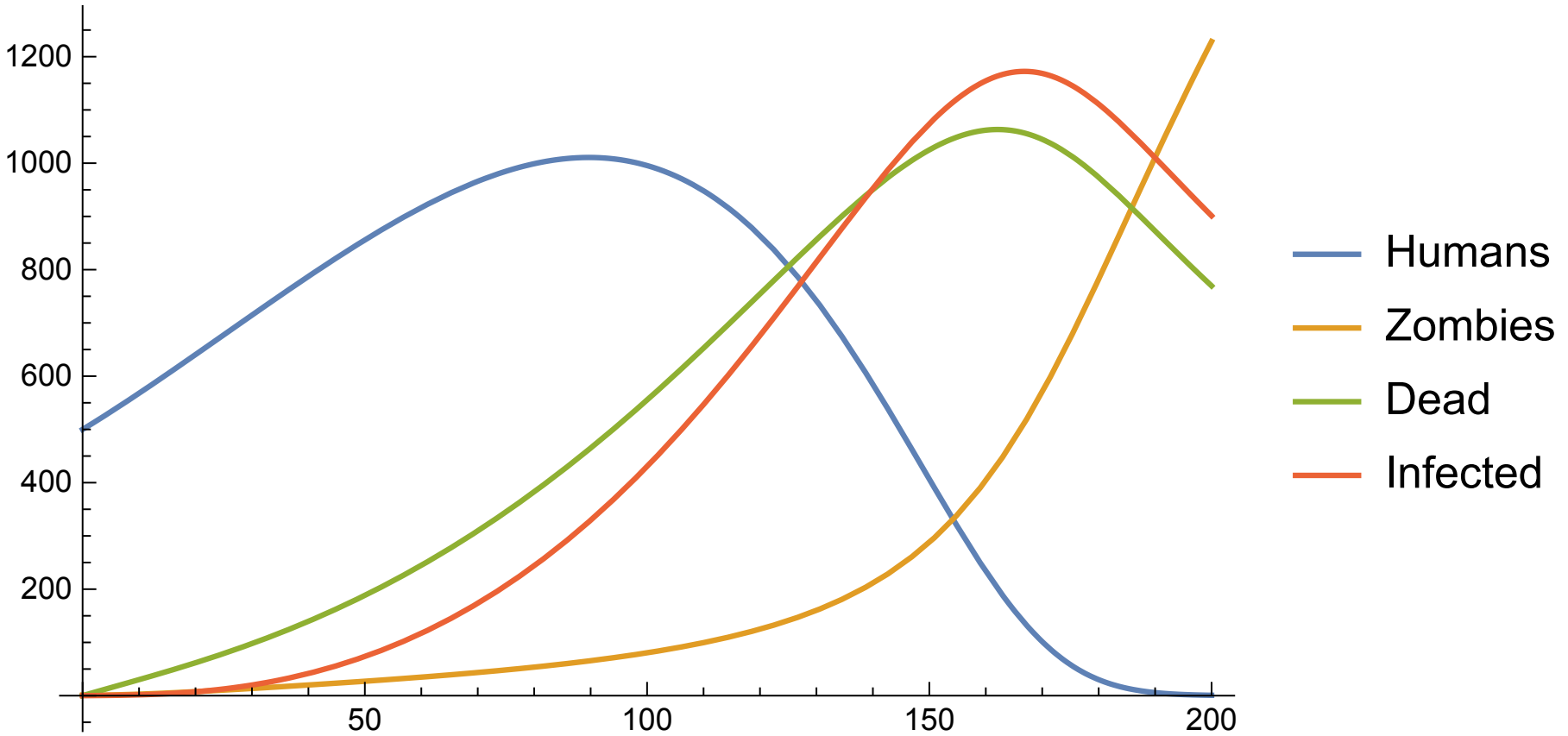


Figure: Model with Latency

With a new infected class, the zombie takeover is simply delayed by approximately 50 days.

Quarantine of Zombies

Under quarantine, zombies will be "closed off" from the outside world (trapped in a cow field, barn, etc..) Unless they escape (parameter θ), no new humans will become infected.

$$H' = \chi H - \gamma HZ - \zeta H$$
$$I' = \gamma HZ - \phi I - \zeta I - \omega I$$
$$Z' = \phi I + \delta D - \kappa HZ - \lambda Z$$
$$D' = \zeta H + \zeta I + \kappa HZ - \delta D + \theta Q$$
$$Q' = \omega I + \lambda Z - \theta Q$$

$\omega =$ Infected people who are no longer able to spread disease (.004).
$\lambda =$ Zombies who can no longer spread disease (.003).
$\theta =$ The chance of zombies or infected people escaping and dying (.006).

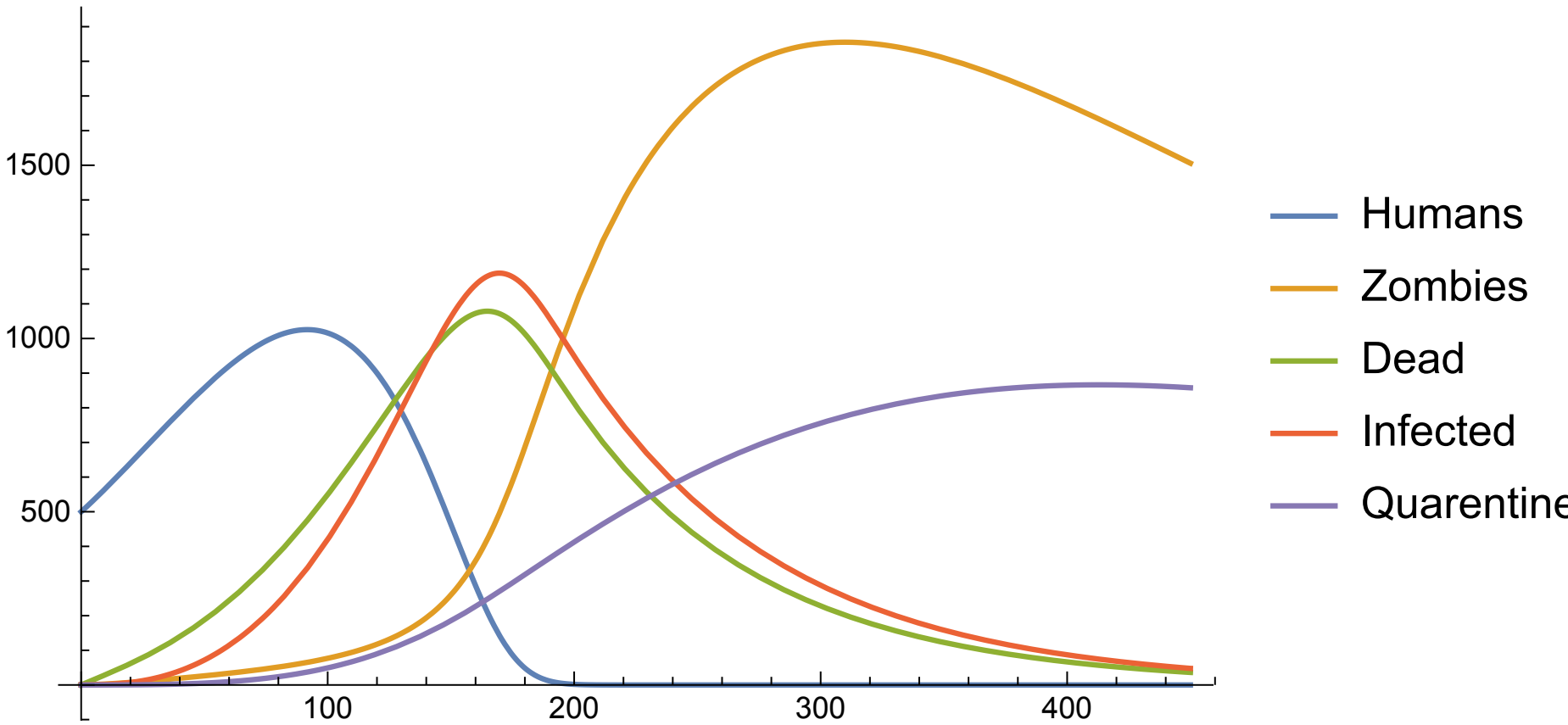


Figure: Model with Quarantine

The growth of the zombie class is delayed due to quarantine, but the human population is ultimately overwhelmed due to an infected class takeover.

Treatment of Zombies

A cure has been found! However, zombies can only turn back into humans; there is no vaccine to prevent humans from becoming zombies. Because there is a cure, there is no longer a need for the quarantine equation. With a treatment, our equations become:

$$H' = \chi - \gamma HZ - \zeta H + cZ$$
$$I' = \gamma HZ - \phi I - \zeta I$$
$$Z' = \phi I + \delta D - \kappa HZ - cZ$$
$$D' = \zeta H + \zeta I + \kappa HZ - \delta D$$

Where cZ represents the cured zombies

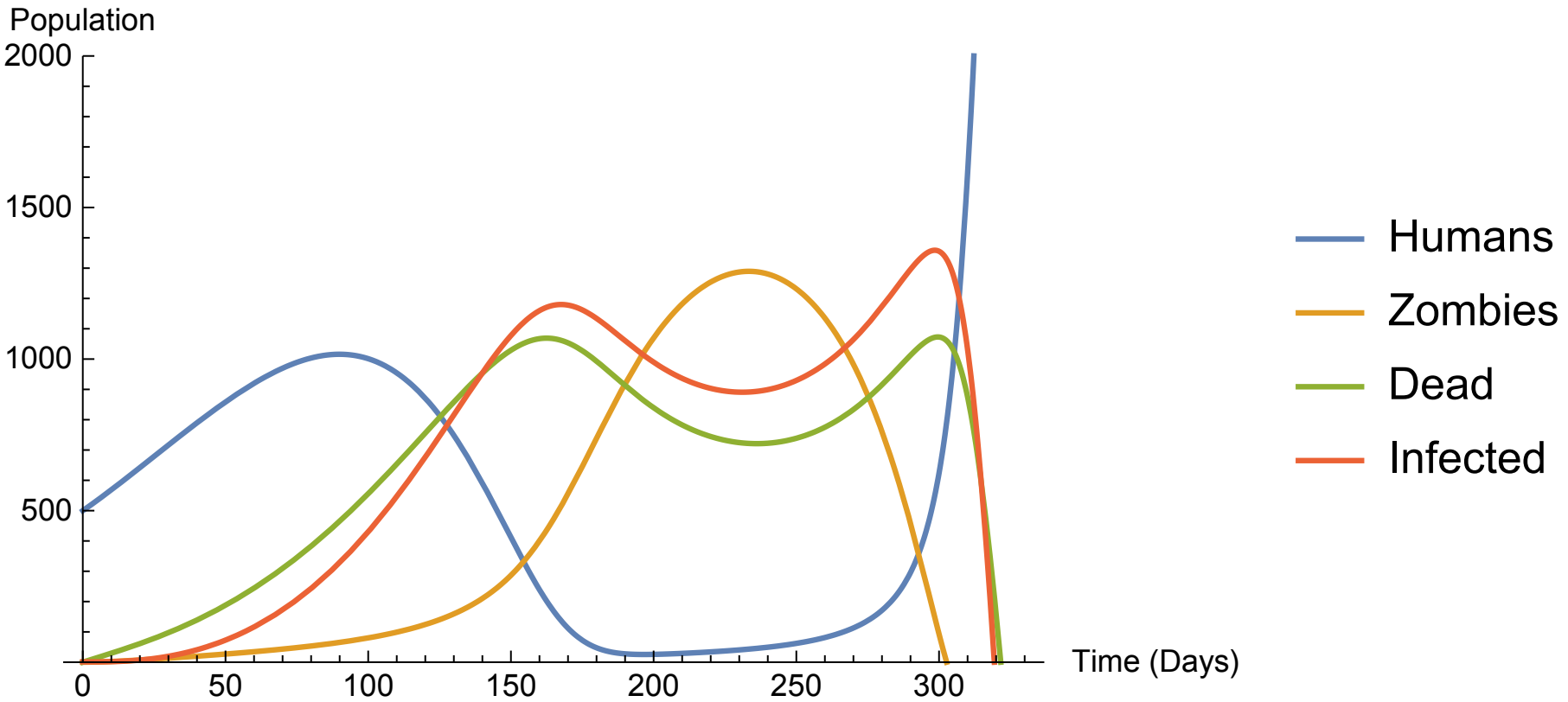


Figure: Model with Treatment after 150 Days

Human Resistance

Humans will undoubtedly fight back against zombies if a zombie outbreak ever occurs. To factor this into our equations, we change κ to be a function of time.

$$H' = \chi H - \gamma HZ - \zeta H$$
$$I' = \gamma HZ - \phi I - \zeta I$$
$$Z' = \phi I + \delta D - \kappa HZ$$
$$D' = \zeta H + \zeta I + \kappa HZ - \delta D$$

The humans will learn how to kill zombies more efficiently over time. $\kappa = (.0000029t)$.

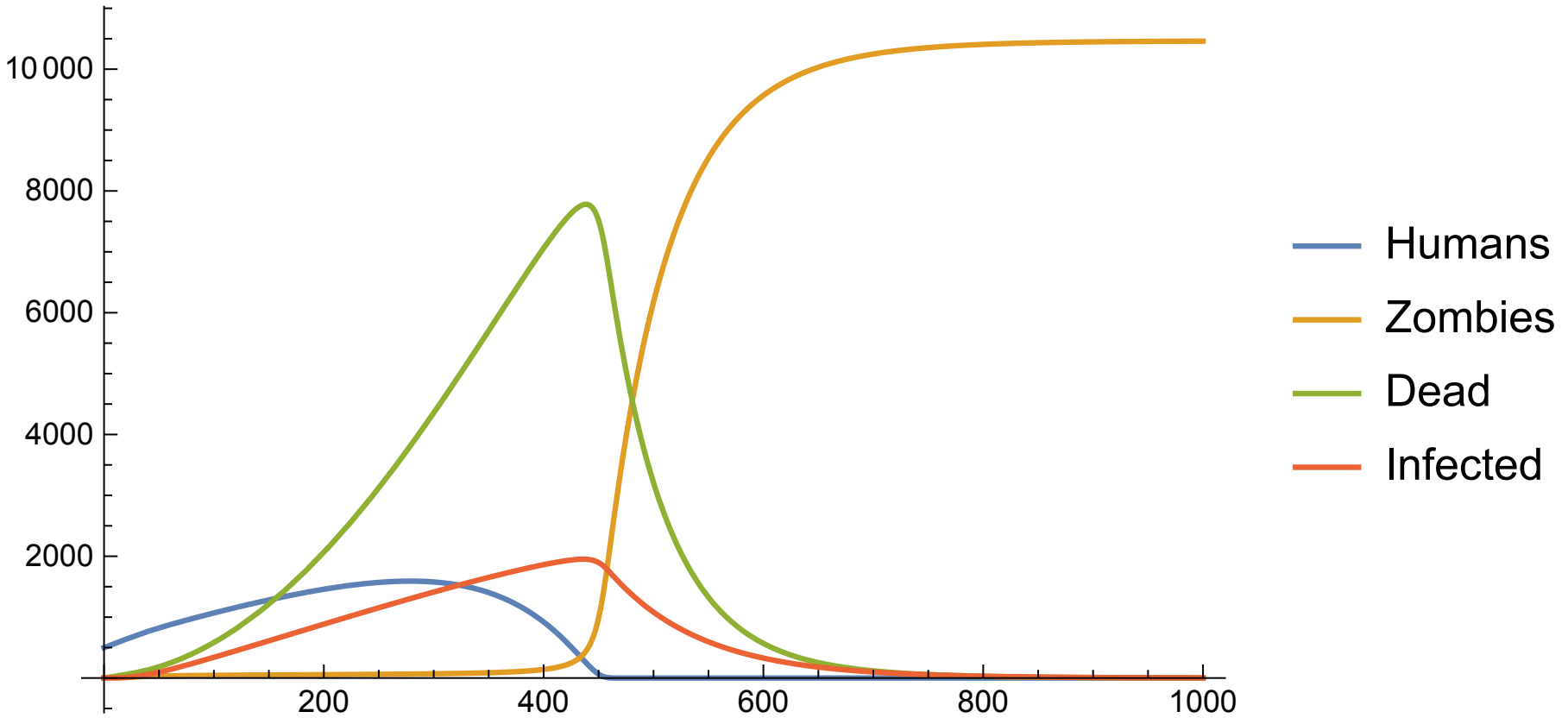


Figure: Model with Latency and Intelligent Resistance

As you can see, the humans will still get overrun by zombies and die.

The following is the result of adding intelligent human resistance to the basic model (our first model). κ is still a function of time.

$$H' = \chi H - \gamma HZ - \zeta H$$
$$Z' = \gamma HZ + \delta D - \kappa HZ$$
$$D' = \zeta H + \kappa HZ - \delta D$$

$\kappa = (.00000013t^2)$

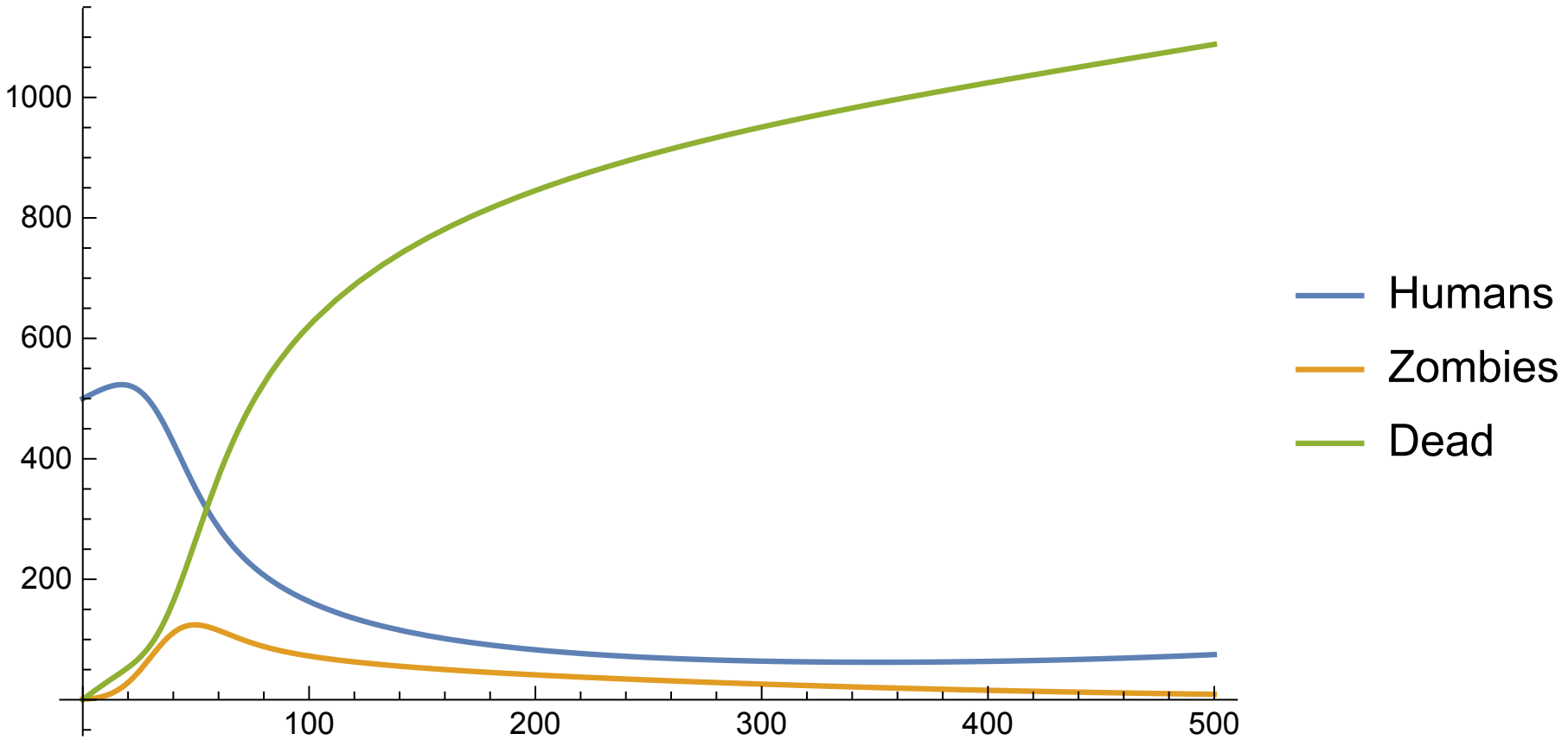


Figure: Basic Equation with Intelligent Human Resistance

References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